MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level

List MF27

LIST OF FORMULAE

AND

RESULTS

for Mathematics and Further Mathematics

For use from 2025 in all papers for the H1, H2 and H3 Mathematics and H2 Further Mathematics syllabuses.

This document has **8** pages.

Singapore Examinations and Assessment Board

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Algebraic series

Binomial expansion:

$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \binom{n}{3} a^{n-3}b^3 + \ldots + b^n$, where n is a positive integer

and $\binom{n}{r} = \dfrac{n\,!}{r\,!(n-r)!}$

Maclaurin expansion:

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^n}{n!}\mathrm{f}^{(n)}(0) + \ldots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (\,|\,x\,| < 1)$$

$$\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{(all } x\text{)}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1} x^r}{r} + \ldots \qquad (-1 < x \leqslant 1)$$

Partial fractions decomposition

Non-repeated linear factors:

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)}$$

Repeated linear factors:

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2}$$

Non-repeated quadratic factor:

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(x^2+c^2)}$$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A \equiv 2 \sin A \cos A$$
$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$
$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Principal values:

$$-\tfrac{1}{2}\pi \leqslant \sin^{-1}x \leqslant \tfrac{1}{2}\pi \qquad (|x| \leqslant 1)$$
$$0 \leqslant \cos^{-1}x \leqslant \pi \qquad (|x| \leqslant 1)$$
$$-\tfrac{1}{2}\pi < \tan^{-1}x < \tfrac{1}{2}\pi$$

Derivatives

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$

Integrals

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{x-a}{x+a}\right)$	$(x > a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{a+x}{a-x}\right)$	$(\lvert x\rvert < a)$
$\tan x$	$\ln(\sec x)$	$(\lvert x\rvert < \frac{1}{2}\pi)$
$\cot x$	$\ln(\sin x)$	$(0 < x < \pi)$
$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x + \cot x)$	$(0 < x < \pi)$
$\sec x$	$\ln(\sec x + \tan x)$	$(\lvert x\rvert < \frac{1}{2}\pi)$

Vectors

The point dividing AB in the ratio $\lambda : \mu$ has position vector $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product:

$$\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

Applications of definite integrals

Arc length of a curve defined in cartesian form: $\displaystyle\int_a^b \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$

Surface area of revolution about the x-axis for a curve defined in cartesian form:

$$\int_a^b 2\pi y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$$

Functions of two variables

Quadratic approximation of f at (a, b):

$$\mathrm{f}(x, y) \approx \mathrm{f}(a, b) + \mathrm{f}_x(a, b)(x-a) + \mathrm{f}_y(a, b)(y-b) + \frac{1}{2}\mathrm{f}_{xx}(a, b)(x-a)^2 + \mathrm{f}_{xy}(a, b)(x-a)(y-b) + \frac{1}{2}\mathrm{f}_{yy}(a, b)(y-b)^2$$

Numerical methods

Trapezium rule (for single strip): $\int_a^b \mathrm{f}(x)\mathrm{d}x \approx \frac{1}{2}(b-a)\left[\mathrm{f}(a)+\mathrm{f}(b)\right]$

Simpson's rule (for two strips): $\int_a^b \mathrm{f}(x)\mathrm{d}x \approx \frac{1}{6}(b-a)\left[\mathrm{f}(a)+4\mathrm{f}\left(\frac{a+b}{2}\right)+\mathrm{f}(b)\right]$

The Newton-Raphson iteration for approximating a root of $\mathrm{f}(x) = 0$:

$$x_2 = x_1 - \frac{\mathrm{f}(x_1)}{\mathrm{f}'(x_1)},$$

where x_1 is a first approximation.

Euler Method with step size h:

$$y_2 = y_1 + h\mathrm{f}(x_1, y_1)$$

Improved Euler Method with step size h:

$$u_2 = y_1 + h\mathrm{f}(x_1, y_1)$$
$$y_2 = y_1 + \frac{h}{2}\left[\mathrm{f}(x_1, y_1) + \mathrm{f}(x_2, u_2)\right]$$

Standard discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $\mathrm{Po}(\lambda)$	$\mathrm{e}^{-\lambda}\frac{\lambda^x}{x!}$	λ	λ
Geometric $\mathrm{Geo}(p)$	$(1-p)^{x-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$

Standard continuous distribution

Distribution of X	p.d.f.	Mean	Variance
Exponential	$\lambda\mathrm{e}^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

Sampling and testing

Unbiased estimate of population variance:

$$s^2 = \frac{n}{n-1}\left(\frac{\Sigma(x-\bar{x})^2}{n}\right) = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$

Regression and correlation

Estimated product moment correlation coefficient:

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\left\{\Sigma(x-\bar{x})^2\right\}\left\{\Sigma(y-\bar{y})^2\right\}}} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)}}$$

Estimated regression line of y on x:

$$y - \bar{y} = b(x - \bar{x}), \quad \text{where } b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

WILCOXON SIGNED RANK TEST

P is the sum of the ranks corresponding to the positive differences,
Q is the sum of the ranks corresponding to the negative differences,
T is the smaller of P and Q.

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of *T*

	Level of significance			
One	0.05	0.025	0.01	0.005
Two	0.1	0.05	0.02	0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

AM-GM inequality:
For any nonnegative real numbers $x_1, x_2, \ldots, x_n$,

$$\frac{x_1 + x_2 + \ldots + x_n}{n} \geqslant \sqrt[n]{x_1 x_2 \ldots x_n},$$

where the equality holds if and only if $x_1 = x_2 = \ldots = x_n$.

Cauchy-Schwarz inequality:
For any real numbers $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$,

$$\left(\sum_{i=1}^{n} u_i v_i\right)^2 \leqslant \left(\sum_{i=1}^{n} u_i^2\right)\left(\sum_{i=1}^{n} v_i^2\right),$$

where the equality holds if and only if there exists a nonzero constant k such that $u_i = kv_i$ for all $i = 1, 2, \ldots, n$.

Triangle inequality:
For any real numbers $x_1, x_2, \ldots, x_n$,

$$|x_1 + x_2 + \ldots + x_n| \leqslant |x_1| + |x_2| + \ldots + |x_n|,$$

where the equality holds if $x_1, x_2, \ldots, x_n$ are all nonnegative.

Inclusion-Exclusion Principle:
For any subsets $A_1, A_2, \ldots, A_n$ of a set,

$$\begin{aligned} |A_1 \cup A_2 \cup \ldots \cup A_n| = {} & |A_1| + |A_2| + \ldots + |A_n| \\ & - [|A_1 \cap A_2| + |A_1 \cap A_3| + \ldots + |A_{n-1} \cap A_n|] \\ & + [|A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \ldots + |A_{n-2} \cap A_{n-1} \cap A_n|] \\ & \quad \vdots \\ & + (-1)^{n-1} |A_1 \cap A_2 \ldots \cap A_{n-1} \cap A_n| \end{aligned}$$

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